

# The “fingers” of the physics Les “doigts” de la physique

Luigi Foschini

Institute TeSRE–CNR, Via Gobetti 101, I-40129, Bologna (Italy)

Email: [foschini@tesre.bo.cnr.it](mailto:foschini@tesre.bo.cnr.it)

February 2, 2008

## Abstract

The passage of particles through matter is one of the principal ways to investigate nature. In this article, we would like to outline the most important stages in the development of the theory about the stopping power.

Le passage des particules à travers de la matière c'est une des routes principales pour étudier la nature. In cet article nous voulons tracer les stages principales de le développement des théorie à propos du pouvoir d'arrêt.

PACS-01.65.+g History of science.  
PACS-34.50.Bw Energy loss and stopping power.

## 1 Introduction

To investigate nature man has always used means which could explore the world beyond human perception. Among these methods, the study of the passage of particles and radiations through matter has been an extremely thriving sector and one of basic importance for modern physics; moreover, it

has put to the test the minds of the greatest scientists of this century: Bohr, Bethe, Fermi, just to quote some of them, for the list would be too long.

The knowledge of the interactions that take place in the passage of particles and radiations through matter, allowed possible to develop several detectors, on whose characteristics the success of an experiment may depend or not. From the analysis of this phenomena, many branches of physics and astrophysics have originated; for example the study of matter in its elementary components, the study of cosmic rays, not to speak about the several applications in the biomedical field.

The main sector of application of these studies is, without doubt, particles physics. It is enough to think about the impact that Wilson's chamber had in modern physics: we cannot observe a particle, but, at least, we can see the effects of its passage. In addition, the studies on the interactions among matter, particles and radiations allowed, since the X-rays discovery, the development of today necessary diagnostic tools. The accession of new detectors, along with the extraordinary development of electronics, allowed us to obtain fundamental results in medicine: just think about the positron emission tomography (PET), the digital radiography, the free electrons laser [1].

From the moment we decide to use these artificial "fingers" in order "to touch" a world out of our ordinary experience, questions of the utmost importance arise. For example, what is the logical correlation between the lightning in a scintillator and the passage of a proton? Is it possible to gain informations from this lightning? What kind of informations? After all, the most important question is: what do we measure, what do we observe?

The literature on this theme is practically boundless, because of the importance of these studies in modern physics. We are not trying to make a complete analysis of the theme here, for this would need at least a book: we want to give an outline of the most important stages of the evolution of the stopping power formula and invite the reader to consult the wide bibliography for a more accurate investigation.

It is worth noting that, in order to improve the readability, the symbols used in this paper will be listed in the Appendix.

## 2 The XIX Century

Even though this story begins at the dawn of the XX century, its roots are to be found in that extraordinary period of discoveries and contradictions that characterized the end of the XIX century. In 1873, the scientific community was upset by the publication of the *Treatise on electricity and magnetism* by J.C. Maxwell, principally because not many had the necessary knowledge to understand the work of the great Scottish scientist. The first experimental evidence of Maxwell's theories came in 1886-1887 only, by H. Herz, who could make experiments on the electromagnetic waves propagation. Nevertheless, many questions remained unsolved: in 1885, J.J. Thompson wrote that he still did not understand what Maxwell called "quantity of electricity" [2].

For this researches, an extremely prolific sector was that of electric discharges in gases, principally began by Faraday in 1836 [3]. However, the first valid results were reached only later, thanks to a quick development of the vacuum technology and to the invention in 1851 of the first rudimental transformer, the Ruhmkorff coil: the device allowed to obtain electrical discharges at very high voltage. The pioneers of this sector were Julius Plücker, Willhelm Hittorf, Eugen Goldstein and Charles Varley [4]. In 1879, it was Goldstein to propose the name of *cathode rays* to explain the phenomena that take place in the electric discharge in extremely rarefied gases. The question on the nature of these rays appeared immediately particularly muddled and the physicists of the time were divided between those who were for the undulatory hypothesis (aether waves) and those who were more inclined for a corpuscular nature. A further contribution came from William Crookes, who could make a whirl spin by striking it with cathode rays [4]. Moreover, he demonstrated that the action of a magnet could deviate these rays. Therefore, the balance of science began to be inclined for the corpuscular hypothesis, even though Heinrich Herz's shrewdness played an important role in the undulatory hypothesis.

The crucial stroke came by Jean Perrin [5] and, above all, by Joseph John Thomson, director of the Cavendish laboratory in Cambridge. He demonstrated definitively the corpuscular hypothesis, by succeeding in measuring the ratio between the mass and the charge of a particle, to which the name of *electron* was given (it had been minted by G.J. Stoney some years before) [6]. It is curious to note that, years later J.J. Thomson's son, George Paget, will become famous with the demonstration of the electron undulatory properties.

In those years other discoveries conquered the world scene: the X-rays were discovered in 1895 by Wilhelm C. Röntgen and radioactivity was discovered in 1896 by Henry Becquerel. Physics began the new century with two kind of particles, atoms and electrons, and with new radiations (X-rays, radioactivity) and, above all, with many question marks.

### 3 Rutherford and Bohr

In September 1895, Ernest Rutherford, a young New Zealander with many hopes and little money, arrived in Cambridge. Thomson's young assistant recognized two different kinds of radiations emitted by uranium and called them  $\alpha$  and  $\beta$ , because of the different absorption in matter. In the following years, Rutherford continued his researches on the nature of these rays in Montreal and returned to England only in 1907 for a chair at Manchester University. In his list of possible researches, the diffusion of the  $\alpha$ -rays was included: this lead to the discovery of the atomic nucleus.

However, Rutherford's highest contribution was to change the linguistic approach to the problem by putting in the foreground issues as collision, scattering and passage of particles through matter [7]. To this purpose the 1905 article is particularly interesting: it suggested a theory on the origin of the scintillation of zinc sulfide exposed to  $\alpha$  radiations. Becquerel thought that this was due to the cleavage of crystals bombarded by  $\alpha$  particles, while Rutherford did not write about a direct effect, but about a secondary process, the recombination of ions produced during the radiation passage.

In the first edition of his book *Conduction of electricity through gases*, J.J. Thomson had hinted to some issues regarding the ionization produced by X-rays and by Bequerel's radiations, nonetheless basing himself upon the kinetic theory of perfect gases (diffusion of a fluid in matter) [8]. In the last chapter only, he considered the problem of what will happen when a charged particle is stopped. In the second edition of the book, published in 1906 [9], Thomson wrote a first theoretical approach to the scattering of two charged particles, elaborated again in a later article [10]. Nevertheless, he remained tied to the intuitive conception according to which the collision produced very small variations on the projectile trajectory; therefore the projectile passed through matter as if it were in a tunnel bouncing on the walls (multiple scattering).

This theory was discussed by Hans Geiger with the help of Ernest Mars-

den, a freshman: between 1908 and 1910 they published together a series of articles, showing the existence of a diffusion of  $\alpha$  particles [11], [12]. Even though the particles reflected with angles wider than  $90^\circ$  were a small percentage (1 over 8000):

... it seems surprising that some of the  $\alpha$  particles, as the experiment shows, can be turned within a layer of  $6 \cdot 10^{-5}$  cm of gold through an angle of  $90^\circ$ , and even more [12].

Therefore, Rutherford's surprise in seeing those results was great: spurred by those results, the New Zealander scientist published, in 1911, his famous essay, where he described the theory on the atoms structure: a nucleus surrounded by electrons [13]. Moreover, he explained the reason why  $\alpha$  particles were reflected with wide angles, introducing the concept of a strong nuclear field due to the concentration of positive electric charge in an extremely small area, nearly punctiform (dimensions  $\sim 10^{-14}$  m).

The year after, a young Danish named Niels Bohr arrived at Manchester, after a disappointing experience with Thomson at Cambridge. Bohr run across the problem of the  $\alpha$  rays energy loss while passing through matter. While he was waiting for some radium to go on with an experimental job, he was attracted by Charles Galton Darwin, grandson to the great Charles Robert, who was in Manchester at the time [14]. Rutherford made Darwin study the theory of the  $\alpha$ -rays penetration in matter, for Darwin had a degree in Applied Mathematics [15]. Bohr understood that the difficulties the mathematician met were due to the two possible hypothesis on the electrons distribution in the atom that Darwin had treated (in the volume of a sphere centered in the nucleus or on the surface of that sphere) and decided to treat the atomic electrons as oscillators. In the meanwhile, Thomson was carrying on researches in this same field, considering an atom with free electrons and neglecting the results obtained in Manchester [16].

It was Niels Bohr who confirmed the validity of Rutherford group's approach, with an article ended and published the following year when he went back to Denmark [17]. This essay did not only represent the first sufficiently wide treatment on particles energy loss while passing through matter, but it was also the prelude to the famous quantum theory of the atom, published in the same year [18]. Among the most interesting points Bohr underlined there were: the necessity of treating the collisions between atoms,  $\alpha$  and  $\beta$  particles in the same way (apart from the obvious differencies of mass and charge);

the time of collision and therefore a first link with the electromagnetic theory of dispersion; the evidence, nearly sure, of the fact that hydrogen has one electron and helium has two. In 1915, Bohr published a second essay where he exposed with greater details the theory of dispersion and considered the relativistic effects [19].

The particles slowing down in the passage through matter remained an interest of Bohr for his whole life, so that after the discovery of the nuclear fission the Danish physicist dedicated himself to the study of the passage of fission fragments in matter [20], [21]. His last work was written in 1954 with Jens Lindhard and regards in particular the capture or loss of electrons by fission fragments in the passage through matter [22].

Bohr's approach is based upon classic mechanics: he considers a heavy particle that passes at such a distance to interact with one electron of one atom of the material, initially considered at rest, thus defining the impact parameter  $b$ . The importance of this parameter consists in separating near and distant collisions, that have therefore a different behaviour [21]. The collisions at distances  $d > b$  can be treated as electromagnetic excitations of harmonic oscillators charged in an electric field, uniform in space, due to the passage of the incident particle. On the other hand, if  $d < b$ , collisions can be treated as scattering of free electrons by the incident particle. Choosing  $b \ll \gamma v / \omega$ , that corresponds to the limit of the impact parameter beyond which the energy transfer is not efficient, and  $b \gg |z|e^2 / mv^2\gamma$ , that can be seen as a dimension of the scattering centre, the energy lost in the collisions is given by the formula:

$$-\frac{dE}{dx} = \frac{4\pi Nz^2 e^4}{mv^2} \left\{ \ln \frac{1.123mv^3}{|z|e^2\omega} - \ln(1 - \beta^2) - \frac{\beta^2}{2} + R_i \right\} \quad (1)$$

where  $R_i$  is the function of the impact parameter, though its contribution to Eq. (1) is about 0.01%, giving therefore an indication of the poor sensibility of Eq. (1) regarding the parameter. Moreover, Eq. (1) shows problems in extreme cases, when  $b \rightarrow 0$  or  $b \rightarrow \infty$ .

## 4 Bethe and Bloch

For many years Bohr's works were practically unrivalled, though there was no total agreement on the measures taken, in particular on the  $\alpha$  and  $\beta$  particles. In the twenties, after the birth of quantum mechanics, the deficiencies of

Eq. (1) were regarded as the failure of classic mechanics in the treatment of atomic phenomena and, in particular, to the fact that energy transfers, in this kind of reactions, take place for discrete quantities.

The first news arrived in the thirties, when Hans Bethe faced the problem again by means of the new quantum mechanics [23]. To tell the truth, there had been other attempts in the twenties, though not completely satisfactory. Bethe solved the problem by using Born approximation. This approximation requires that the amplitude of the wave scattered by the atomic electron field must be small when compared to the one of the incident wave (undisturbed). This is expressed with the condition [26]:

$$\frac{ze^2}{\hbar v} \ll 1 \quad (2)$$

Condition Eq. (2) is satisfied in case of high speed and low electric charge of the incident particle. A further simplification came when the speed of the incident particle is considered to be much higher than that of the electrons of the absorbent material atoms:

$$E \gg \frac{M}{m} E_{el} \quad (3)$$

where  $E$  is the energy of the incident particle and  $E_{el}$  is the ionization potential of the atomic electron. Under these conditions and considering non-relativistic speeds the energy loss is:

$$-\frac{dE}{dx} = \frac{4\pi N z^2 e^4}{mv^2} B \quad (4)$$

that gives the stopping power of the material. The term  $B$  (*stopping number*) is expressed as:

$$B = Z \ln \frac{2mv^2}{I} \quad (5)$$

Comparing Eq. (1) to Eq. (4) and Eq. (5) we note that the logarithmic term in Bethe's formula does not depend from the incident particle charge. In the case of relativistic speeds, Eq. (5) becomes [24]:

$$B = Z \left\{ \ln \frac{2mv^2}{I} - \ln(1 - \beta^2) - \beta^2 \right\} \quad (6)$$

It is worth noting that Bethe's and Bohr's formulas have a field of validity each: therefore they can be considered complementary. A synthesis of the two formulas was carried out in 1933 by Felix Bloch [25], who obtained:

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{mv^2} NZ \left\{ \ln \frac{2mv^2}{I} + \psi(1) - \text{Re}\psi \left( 1 + i\frac{ze^2}{\hbar v} \right) \right\} \quad (7)$$

where  $\psi$  is the logarithmic derivative of Euler's  $\Gamma$  function. Eq. (7) becomes Bethe's formula for  $ze^2/\hbar v = 0$ , while it is reduced to Bohr's formula when  $ze^2/\hbar v \gg 1$ . It is worth noting that Eq. (7) is referred to the case of non-relativistic speeds. For  $\beta \rightarrow 1$  it is sufficient to detract the terms  $[\ln(1 - \beta^2) + \beta^2]$  to the quantity enclosed within brace brackets

In 1953, a detailed work by Bethe and Ashkin [26] was added to the excellent review done by Bohr [21]; the new work included also various cards and graphs relatives to experimental measures and it analysed widely any possible anomaly.

## 5 Today

What is today known as the Bethe-Bloch formula is the principal instrument to reckon the energy loss due to particles passage through matter. Several correction coefficients were added to consider many effects [27], nevertheless the Bethe-Bloch formula still remains the starting point.

A first correction was signaled by Bhabha [28] and regarded the maximum energy that can be transferred to an electron, initially at rest, in a free collision, that results:

$$T_{max} = \frac{2mc^2\beta^2\gamma^2}{1 + 2\gamma\frac{m}{M} + (\frac{m}{M})^2} \quad (8)$$

and therefore takes the place of the simple  $2mv^2$  in equation Eq. (5).

Another important corrective factor is the so called "density effect", due to the fact that the electric field of the incident particle has a tendency to polarize atoms. As a matter of facts, Bohr, Bethe and Bloch's analysis regarded gases at low density; these gases can be seen as an aggregate of independent particles, which does not happen in solid bodies or in liquid ones. In the fourties, Enrico Fermi noticed that the energy loss did not depend from physical parameters only, those parameters Bethe and Bloch took account of, though it depends also from the material dielectric properties

[29]. It is well worth to remember that in 1924 Fermi had already noticed the analogy between the collision and dispersion phenomena and had proposed to evaluate the stopping power by basing himself upon the empirical evidence of the high frequency radiation absorption [30]. To evaluate the density effect Fermi used classical electrodynamics and a simplified model of the atom; later on, Gian Carlo Wick [31] and, independently (the time gap was due to the war), Otto Halpern and Harvey Hall [32], used a more complicated model, a multifrequencies one that takes account of the dispersion; moreover Halpern and Hall considered also the damping of the conduction electrons. An empirical expression of the density correction for a wide range of materials was given by Sternheimer [33]. Later on, Sternheimer himself along with Peierls, gave a generalized version of it [34].

However, the classical and semiclassical approach to density effect was readily substituted by the quantum one [35], [36], even though it did not disappear completely. In 1960, Landau and Lifshitz still gave a semiclassical version of it in their famous *Course of Theoretical Physics* [37] and in 1980 Ahlen wrote that this kind of approach may still be satisfactory, if it is used in energy intervals where polarization effects are not prevalent [27]. Between the macroscopic effect and the microscopic one, another way has to be signaled; it was followed by Aage Bohr to try and evaluate the relativistic effects, above all [38]. These are the cause of discordance between the classic density correction and the quantum one: as a matter of fact it is necessary to make a distinction between a longitudinal contribution and a transversal one for the first only is present in the classic derivation [35]. The transversal term puts on a significant probability at relativistic speeds and is to be held responsible for Čerenkov radiation.

Another problem in the Bethe-Bloch formula is made up by the mean excitation potential of the absorbing material ( $I$ ), which is function of  $Z$ . Being it an average value it is possible to mantain the computation at a moderate accuracy level, which is an advantage. On the other hand, in materials composed by various elements and in particular in organic materials, the chemical bond and other aggregation characteristics may insert sensitive variations through changes in the spectrum of the excited levels [36].

Another assumption effectuated in the Bethe-Bloch formula is that the incident particle is decidedly faster than atomic electrons as specified in equation Eq. (3). Nevertheless, as the incident particle speed decreases, the contribution to the stopping from the inner shell electrons ( $K, L, \dots$ ) disappears. It is therefore necessary to insert a further correction coefficient to take this

phenomenon into account [39], [36].

Moreover, there may be differences in conformity with the incident particle, if it is positive or negative. It is the so called “Barkas effect”, discovered in the 50’s studying the decay of  $K \rightarrow 3\pi$ : it was observed that the  $\pi^-$  particles had slightly longer paths than the  $\pi^+$  ones [40]. A further correction coefficient is then introduced and it depends, according to the particle, on  $z^3$  or  $z^4$  [41].

For other correction coefficients or changes to be introduced in the Bethe-Bloch formula according with its use, we would suggest to read the wide reviews listed in the bibliography, in particular [26], [27].

Lately, the only news of a certain prominence is the theory by Jens Lindhard, a Bohr’s scholar, and Allan Sørensen, relative to the energy loss of heavy ions at relativistic speeds; this theory starts from the well-known Bethe-Bloch formula [42]. The authors notice that the deviations that take place in the case of relativistic ions are to be expressed in terms of cross section for the scattering of an electron, free from the Coulomb field of ions; the authors carry out the necessary computations using Dirac equation for relativistic ions.

## 6 Conclusions

In 1998, the *Particle Data Group* relates this expression [27]:

$$-\frac{dE}{dx} = \frac{4\pi N r_e^2 m c^2 z^2 Z}{A \beta^2} \left[ \frac{1}{2} \ln \left( \frac{2mc^2 \beta^2 \gamma^2 T_{max}}{I^2} \right) - \beta^2 - \frac{\delta}{2} \right] \quad (9)$$

which is adaptable to the majority of cases. Some correction coefficients - already discussed above - follow. As we can see the Bethe-Bloch formula is still today a “finger” of physics, perhaps the most important one. Maybe, it is a botched up one but is still very efficient and it is a necessary instrument not only for the understanding of the world around us though also for its technological applications. It is well worth to think about the use of radiations in cancers treatment, a therapy that requires a great caution to avoid telling the famous joke: “the operation was successfull, but the patient died”.

## 7 Acknowledgements

A sincere thank to Federico Palmonari for the critical revision of the text and for his useful suggestions. A special thank to the Library of the Department of Physics (University of Bologna), without which this research would not be possible.

## Appendix

List of symbols:

$Ze$ : electric charge of the absorbing material;  
 $ze$ : electric charge of the incident particle;  
 $A$ : atomic mass of the absorbing material;  
 $M$ : rest mass of the incident particle;  
 $m$ : electron rest mass;  
 $r_e$ : electron classical radius;  
 $v$ : incident particle speed;  
 $c$ : speed of light in vacuum;  
 $\beta = v/c$ ;  
 $\gamma = (1 - \beta^2)^{-1/2}$ ;  
 $\delta$ : density correction;  
 $N$ : number of electron for absorbing material volume unit;  
 $d$ : distance;  
 $b$ : impact parameter;  
 $I = \hbar\omega$ : mean excitation energy of absorbing material.

## References

- [1] Jean B. 1997 *Nucl. Instrum. Methods A* **393** 540-543; Ott R.J. 1997 *Nucl. Instrum. Methods A* **392** 396-401.
- [2] Bellone E. 1990 *Caos e armonia - Storia della fisica moderna e contemporanea* (UTET: Torino) p. 255.
- [3] Faraday M. 1933 *Faraday's diary - Being the various philosophical notes of experimental investigation - vol. 3* ed. T. Martin (G.Bell and Sons: London) pp. 14 and following.
- [4] See, for example, Bellone E. 1990 *Caos e armonia - Storia della fisica moderna e contemporanea* (UTET: Torino); Segrè E. 1980 *From X-Rays to Quarks* (W.H. Freeman: New York).
- [5] Perrin J. 1895 *Compt. Rend. Acad. Sci.* **121** 1130-1134.
- [6] Thomson J.J. 1897 *Philos. Mag.* **44** 293-316.
- [7] Rutherford E. 1905 *Philos. Mag.* **10** 163-176; Rutherford E. 1906 *Philos. Mag.* **11** 166-176; Rutherford E. 1906 *Philos. Mag.* **12** 134-146.
- [8] Thomson J.J. 1903 *Conduction of electricity through gases* (Cambridge University Press: Cambridge).
- [9] Thomson J.J. 1906 *Conduction of electricity through gases* 2nd rev. ed. (Cambridge University Press: Cambridge) pp. 370-382.
- [10] Thomson J.J. 1910 *Proc. Cambridge Philos. Soc.* **15** 465-471.
- [11] Geiger H. 1908 *Proc. R. Soc. A* **81** 174-177; Geiger H. 1910 *Proc. R. Society A* **83** 492-504.
- [12] Geiger H. and Marsden E. 1909 *Proc. R. Soc. A* **82** 495-500.
- [13] Rutherford E. 1911 *Philos. Mag.* **21** 669-688.
- [14] Pais A. 1993 *Niels Bohr's Time: in physics, philosophy, and polity* (Oxford University Press: Oxford); italian translation *Il Danese tranquillo - Niels Bohr, un fisico e il suo tempo 1885-1962*, (Bollati-Boringhieri: Torino) pp. 132-134.

- [15] Darwin C.G. 1912 *Philos. Mag.* **23** 901-920.
- [16] Thomson J.J. 1912 *Philos. Mag.* **23** 449-457.
- [17] Bohr N. 1913 *Philos. Mag.* **25** 10-31.
- [18] Bohr N. 1913 *Philos. Mag.* **26** 476-502.
- [19] Bohr N. 1915 *Philos. Mag.* **30** 581-612.
- [20] Bohr N. 1940 *Phys. Rev.* **58** 654-655; Bohr N. 1941 *Phys. Rev.* **59** 270-275; Bohr N., Bøggild J.K., Brostrøm K.J. and Lauritsen T. 1940 *Phys. Rev.* **58** 839-840.
- [21] Bohr N. 1948 *K. Danske Vidensk. Selskab. Math. fys. Meddel.* **18** 1-144.
- [22] Bohr N. and Lindhard J. 1954 *K. Danske Vidensk. Selskab. Math. fys. Meddel.* **28** 1-31.
- [23] Bethe H. 1930 *Ann. Physik* **5** 325-400.
- [24] Bethe H. 1932 *Zeit. Phys.* **76** 293-299; Møller C. 1932 *Ann. Physik* **14** 531-585.
- [25] Bloch F. 1933 *Ann. Physik* **16** 285-320; Bloch F. 1933 *Zeit. Phys.* **81** 363-376.
- [26] Bethe H. and Ashkin J. 1953 in *Experimental Nuclear Physics - vol. 1* ed. E. Segrè (Wiley: New York) pp. 166-357.
- [27] Ahlen S.P. 1980 *Rev. Mod. Phys.* **52** 121-173; Geissel H. 1985 in *Proceedings of the Niels Bohr Centennial Conference: Semiclassical descriptions of atomic and nuclear collisions* ed. J. Bang and J. De Boer (North-Holland: Amsterdam) pp. 431-462; Particle Data Group 1998 *Eur. Phys. J. C* **3** 1-794.
- [28] Bhabha H.J. 1938 *Proc. R. Soc. London A* **164** 257-294.
- [29] Fermi E. 1939 *Phys. Rev.* **56** 1242; Fermi E. 1940 *Phys. Rev.* **57** 485-493.
- [30] Fermi E. 1924 *Zeit. Phys.* **29** 315-327.

- [31] Wick G.C. 1940 *Ricerca Scientifica* **11** 273-274; Wick G.C. 1941 *Ricerca Scientifica* **12** 858-873; Wick G.C. 1943 *Nuovo Cimento* **1** 302-313.
- [32] Halpern O. and Hall H. 1948 *Phys. Rev.* **73** 477-486.
- [33] Sternheimer R.M. 1952 *Phys. Rev.* **88** 851-859.
- [34] Sternheimer R.M. and Peierls R.F. 1971 *Phys. Rev. B* **3** 3681-3692.
- [35] Fano U. 1956 *Phys. Rev.* **102** 385-387; Fano U. 1956 *Phys. Rev.* **103** 1202-1218.
- [36] Fano U. 1963 *Ann. Rev. Nucl. Sci.* **13** 1-66.
- [37] Landau L.D. and Lifshitz E.M. 1960 *Course of Theoretical Physics - vol. 8: Electrodynamics of continuous media* (Pergamon Press: Oxford) pp. 344-359.
- [38] Bohr A. 1948 *K. Danske Vidensk. Selskab. Math. fys. Meddel.* **24** 1-52.
- [39] Bichsel H. 1963 in *American Institute of Physics Handbook* ed. D.E. Gray (McGraw-Hill: New York) pp. 8.20-8.47.
- [40] Barkas W.H., Birnbaum W. and Smith F.M. 1956 *Phys. Rev.* **101** 778-795; Barkas W.H., Dyer J.N. and Heckman H.H. 1963 *Phys. Rev. Lett.* **11** 26-28.
- [41] Lindhard J. 1976 *Nucl. Instrum. Methods* **132** 1-5.
- [42] Lindhard J. and Sørensen A.H. 1996 *Phys. Rev. A* **53** 2443-2456.